Chapter 9 - Integration

Integration is the reverse of differentiation. We can use integration to find areas bounded between a curve and the coordinate axes.

Notation

The ∫ symbol is used to represent integration. Since integration is the reverse of differentiation, we know that:

$$\text{If } \frac{dy}{dx} = f(x) \text{ then } y = \int f(x)\, dx$$

The dx means that we are integrating with respect to x

The integral sign

The expression we want to integrate goes between the integral sign and the dx. This is known as the integrand.

Indefinite integrals

Here, you need to integrate functions of the form x^n, where *n* is a constant and $n \neq -1$. To integrate functions of this form, you can use the following:

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$$

The "+c" is known as the constant of integration. To see why we must add this constant to our result, consider these functions:

$$y = x^2 + 2$$
$$y = x^2$$
$$y = x^2 - 9$$

If we differentiate the above functions, the result is the same: $dy/dx = 2x$ because the constant term disappears upon differentiation. However, since integration is the reverse of differentiation, we should be able to integrate 2x and get back to whichever of those functions we started off with. To allow for this, we have to add the unknown constant of integration, *c*, to the end result. This process is known as indefinite integration.

Definite integrals

A definite integral is one where the integral is bounded between two limits. The main difference between a definite integral and an indefinite integral is that the former will yield a numerical value while the latter will yield a function. To calculate a definite integral:

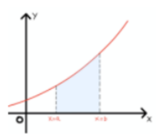
$$\int_a^b f'(x)\, dx = [f(x)]_a^b = f(b) - f(a)$$

Finding Areas

You can use definite integration to find the area bounded between a curve and the x-axis (areas under the line of the curve).

The area between a curve y = f(x), the lines x = a, x = b, and the x-axis is given by:

$$Area = \int_a^b f(x)\, dx$$



Areas under the x-axis

When integrating over an interval where the curve is below the x-axis, the resultant area will be negative. Therefore, extra care must be taken when finding the areas under curves which are not positive.

★ When integrating over an interval where the curve is both above and below the x-axis, you should split the integral up into separate regions where the function is strictly positive or negative in each.