AS MATH - STATISTICS

Chapter 1 - Measure of Central Tendencies

The mean is the sum of all values divided by the number of values
The median is the middle value when values are arranged in order
The mode is the most commonly occurring value

Take the numbers 7, 6, 5, 6, 8, 5, 7, 8, 9

Mean = 68/10 = 6.8
- The sums of all the values is 68 and the total values are 10

Median =
- Order the sequence: 5, 5, 6, 6, 7, 7, 7, 8, 8, 9
- Isolate the middle value: 5.5 — so the 5 and 6 values — hence the answer is 7+7/2 = 7

Mode = 7 as it occurs most frequently

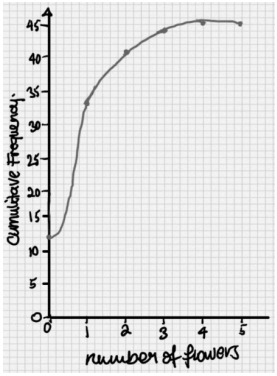
A frequency table is another way of storing data
Frequency is the number of times a value occurs

Number of Flowers	Frequency (f)
0	12
1	21
2	8
3	3
4	0
5	1

From the frequency table we can see that 12 times there were 0 flowers (12 is the frequency), and 21 times there was 1 flower (21 is the frequency)

To find the mean of a frequency table you multiply the value by frequency and add up the products of the frequency and number:

0 x 12 = 0
1 x 21 = 21
2 x 8 = 16
3 x 3 = 9
4 x 0 = 0
5 x 1 = 5



Add up all of the values and you get 51

Divide 51 by the total frequency: 51 / 45 = 1.1 mean value

The cumulative frequency is calculated by adding each frequency from a frequency distribution table to the sum of its predecessors:

12 + 21 = 33
33 + 8 = 41
41 + 3 = 44
44 + 0 = 44

1 = 45

This data is usually represented by a frequency table (as shown above) or a frequency curve.

Quartile and interquartile range

It is the range of values between 25% and 75%

To find the quartile range, find 25% and 75% of the frequency. Draw the line from the frequencies to their corresponding values on the x-axis and subtract them:

Lower quartile = 25/100 x 45 = 11.25
Upper quartile 75/100 x 45 = 33.75
Hence interquartile range will be 1.1 - 0.3 = 0.8

Variance and Standard Deviation

Variance and standard deviation are two other types of central tendencies.
Variance is the **standard deviation** SQUARED
They can be found using the following formula:

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n - 1}$$

Chapter 2 - Representation of data

Data can be represented in many different ways, including:

1. Cumulative frequency graphs (such as the one shown above)

2. Stem and leaf diagrams

For example:

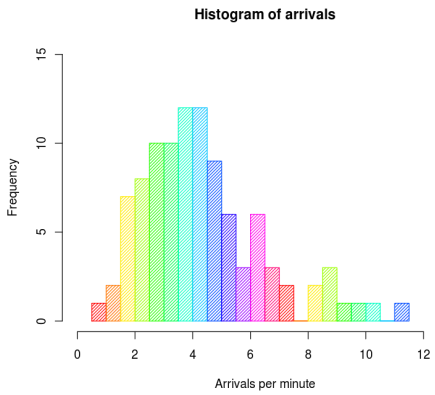
2.3, 2.5, 2.5, 2.7, 2.8 3.2, 3.6, 3.6, 4.5, 5.0

And here is the stem-and-leaf plot:

Stem	Leaf
2	3 5 5 7 8
3	2 6 6
4	5
5	0

Stem "2" Leaf "3" means **2.3**

- In this case, each leaf is a decimal
- It is OK to repeat a leaf value
- 5.0 has a leaf of "0"



3. Histograms

Histograms are used to present continuous data. They show the rough location and general shape of the data and how spread out it is. The area of each bar corresponds to the frequency of each class.

To calculate the height of each bar (frequency density):

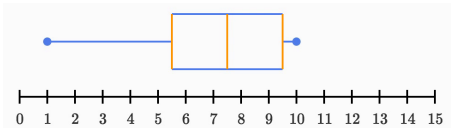
Area of the bar = *k* x frequency

When *k* = 1:

Frequency density = frequency/class width

Joining the top of each bar in a histogram forms a frequency polygon.

4. Box and whisker diagrams (Boxplots)



1. Collect and organize your data.
2. Calculate the median.
3. Calculate each of the quartiles.
4. Create your plot line and draw a line for each of your quartiles.
5. Create a box connecting the quartiles.
6. Find the interquartile range.
7. Find the new upper and lower limits.

Chapter 3 - Probability

★ It is the relative frequency of an event happening

For example:
Getting a 1 when you roll a fair die once = ⅙
This is because the number one only appears once on the die and there are 6 total numbers on the die

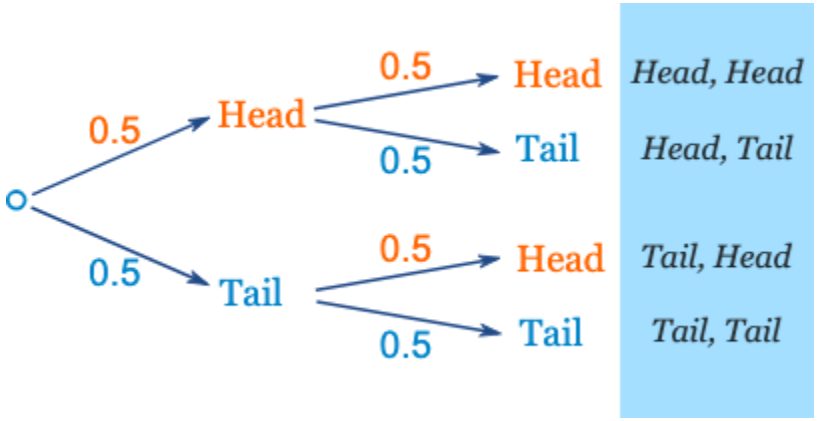
Getting a head when flipping a coin = ½
Because there are two sides to a coin and one head hence it is ½

A simple way to calculate the probability of an event is with the help of a tree diagram

Steps

- Fill in the probabilities on the branches.
- Consider which outcomes are required to answer the question.
- Find the probability of those outcomes by multiplying along the branches.
- Use the probability/probabilities you have calculated to answer the question.

For example, in a tree diagram if you were to flip the coin twice



Getting head twice = ½ x ½ = ¼
Getting tail twice = ¼
Getting diff outcomes on the two throws

1. Head first tial second ½ x ½ = ¼
2. Tail first head second ½ x ½ = ¼

Then you add both of those outcomes

¼ + ¼ = ½

Chapter 4 - Permutations and Combinations

In mathematics, combination and permutation are two different ways of grouping elements of a set into subsets.

Permutation cares about the order
The **combination** doesn't care about the order

Permutation and combination are the ways to select certain objects from a group of objects to form subsets with or without replacement.

In mathematics, **permutation relates to the act of arranging all the members of a set into some sequence or order**

The combination is a way of selecting items from a collection, such that (unlike permutations) the order of selection does not matter.

Permutation formula:

nPr = (n!) / (n-r)!

Combination formula:

$$ {}_nC_r = \binom{n}{r} = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!} $$

Permutation	Combination
Arranging people, digits, numbers, alphabets, letters, and colors	Selection of menu, food, clothes, subjects, and team.
Picking a team captain, pitcher, and shortstop from a group.	Picking three team members from a group.
Picking two favorite colors, in order, from a color brochure.	Picking two colors from a color brochure.
Picking first, second, and third place winners.	Picking three winners.

Example:

How many numbers of four digits can be formed with the digits 1,2,3,4 and 5? (repetition of digits is not allowed)

Required number: ${}^5P_4$ = 5! / 1! = 5 x 4 x 3 x 2 = 120

Combinations:

Selecting 4 fruits out of 10 fruits

$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

$$ {}^{10}C_4 = C(n, r) = C(10, 4) = \frac{10!}{(4!(10-4)!)} = \frac{10!}{4! \times 6!} = 210 \text{ ways} $$

Chapter 5 - Binomial Distribution

The binomial distribution is calculated by multiplying the probability of success raised to the power of the number of successes and the probability of failure raised to the power of the difference between the number of successes and the number of trials.

$$ P_x = \binom{n}{x} p^x q^{n-x} $$

n= number of values
ρ= probability of success
q= probability of failure

$$ n = 7 \text{ shots} $$
$$ p = 0.82 $$
$$ k = 4 \text{ free throws} $$

$$ P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} $$

$$ P(X = 4) = \binom{7}{4} 0.82^4 (1-0.82)^{7-4} $$

$$ P(X = 4) = \left(\frac{7!}{4!(7-4)!}\right) 0.82^4 (0.18)^3 $$

$$ P(X = 4) = (35) 0.82^4 (0.18)^3 = 0.0923 $$

To find the mean of binomial distribution E(X) = n x ρ = nρ
To fid variance it is VAR(X) = n x ρ x q = nρq

Chapoter 6 - Geometric Distribution

A geometric distribution is defined as a discrete probability distribution of a random variable "x" which satisfies some of the conditions.

Conditions

- A phenomenon that has a series of trials
- Each trial has only two possible outcomes – either success or failure
- The probability of success is the same for each trial

$$P(X = x) = \rho \cdot q^{(x-1)}$$

Formula

The main condition of geometric distribution is that it ends after the first successful probability is achieved

$$p = \frac{3}{75} = 0.04 \qquad k = 6$$

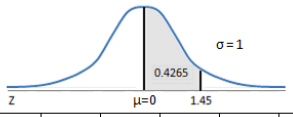
$$P(X = k) = p(1-p)^{k-1}$$
$$P(X = 6) = 0.04(1-0.04)^{6-1}$$
$$P(X = 6) = 0.04(0.96)^{5} = 0.0326$$

To find the mean of the geometric distribution formula is 1/ρ where ρ is the probability of success

Chapter 7 - Normal Distribution

Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score. For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

This is the standard distribution table always given in MF 19 and always required in normal distribution questions

In a normal distribution, data are symmetrically distributed with no skew. Most values cluster around a central region, most tapering off as they go further away from the center. The measures of central tendency (mean, mode, and median) are the same in a normal distribution

Formula

$$Z = \frac{x - \mu}{\sigma}$$

Score, Mean, SD

The value you get from the formula you have to read it corresponding to the table and the value from the table is the probability of the event

For example

Mean = 56
S.d = 7
X = 70

Find the standardized score: (70 - 56)/7 = 2

Then you read the corresponding value at 2.0 on the table
2 on the y-axis and after the decimal, in this case, zero on the x-axis

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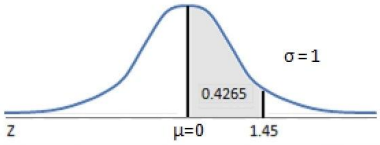
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
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0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
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2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000