# Chapter 1 - Quadratics

## Quadratic equations

An example of a quadratic equation: $f(x) = ax^2 + bx + c$

There are two ways to solve quadratic equations:

1. *Breaking the Middle Term*

Take the equation $6x^2 + 19x + 10$

1) Find the product of the first and last term (a . c)
   $6 \times 10 = 60$

2) Find the factors of 60 in such a way that addition or subtraction of that factors is the middle term ($19x$)
   Two factors of 60 include <u>15</u> and <u>4</u> which, when added together, give 19

3) Splitting of the middle term - ($15 \times 4 = 60$ **and** $15 + 4 = 19$). Write the middle term using the sum of the two new factors, including the proper signs:
   $6x^2 + 15x + 4x + 10$

4) Group the terms to form pairs - the first two terms and the last two terms go together. Factor each pair by finding common factors:
   $3x(2x + 5) + 2(2x + 5)$

5) Factor out the shared (common) binomial parenthesis:
   $(3x + 2)(2x + 5)$

2. *Using the Quadratic Formula*

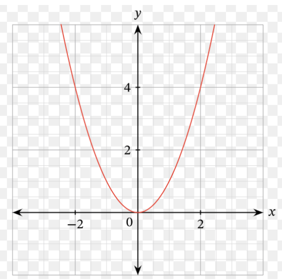
The Quadratic Formula: $-b \pm \sqrt{(b^2-4ac)} / (2a)$

Take the same equation $6x^2 + 19x + 10$

1) Insert the respective values in the formula
   $(-19 \pm \sqrt{19^2 - (4 \times 6)(10)}) / (2 \times 6)$

2) Use the formula to get the answer first with the minus sign then repeat with an addition sign
   $(-19 - \sqrt{19^2 - (4 \times 6)(10)}) / (2 \times 6) = 2/3$
   $(-19 + \sqrt{19^2 - (4 \times 6)(10)}) / (2 \times 6) = -5/2$

## The Quadratic Curve

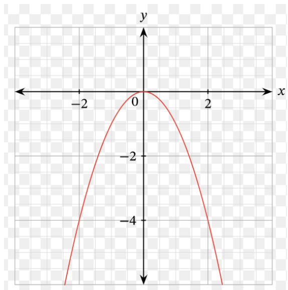



If the coefficient of $x^2$ is +ve then the curve is downward shaped (happy face)

If the coefficient of $x^2$ is -ve then the curve is upward shape (sad face)

> the coefficient is +ve - opens upwards

the coefficient is a -ve - opens downward <




## The "Completing the Square" Method

This method is used for finding the turning point of a graph and helps in plotting a parabola.
The completing the square equation is $n(x - h)^2 + k$

- If $n > 0$, the parabola will have a MINIMUM value (so it will be an upward-facing curve)
- If $n < 0$, the parabola will have a MAXIMUM value (so it will be a downward-facing curve)

In the equation, the point (h,k) is the turning point.

How to get a quartic equation into completing the square

**Step 1:** Write the quadratic equation as $x^2 + bx + c$. (the coefficient of $x^2$ needs to be 1. If not, take it as the common factor.)

**Step 2:** Determine half of the coefficient of $x$.

**Step 3:** Take the square of the number obtained in Step 1.

**Step 4:** Add and subtract the square obtained in Step 2 from the $x^2$ term.

**Step 5:** Factorize the polynomial and apply the algebraic identity $x^2 + 2xy + y^2 = (x + y)^2$ (or) $x^2$, $2xy + y^2 = (x + y)^2$ to complete the square.

Quadratic Inequalities

There are 3 types of quadratic inequalities:

- $ax^2 + bx + c < 0$
- $ax^2 + bx + c > 0$
- $ax^2 + bx + c = 0$

To find the discriminant, use the formula $b^2 - 4ac$

- $ax^2 + bx + c < 0$ has no real roots
- $ax^2 + bx + c > 0$ has two distinct real roots (distinct = different)
- $ax^2 + bx + c = 0$ has one real root