Chapter 8 - Differentiation

Rules of Differentiation

Differentiation of a **scalar multiple** of a function	$\frac{d}{dx}(ay) = a\frac{dy}{dx}$
Differentiation of the **sum/difference** of a function	$\frac{d}{dx}(y_1 \pm y_2) = \frac{dy_1}{dx} \pm \frac{dy_2}{dx}$
Differentiation of **Constant Function** y = c, where c is a constant	$\frac{dy}{dx} = 0$
Differentiation of a **Power Function**, where n is a real number	$\frac{dy}{dx} = nx^{n-1}$

$\frac{dy}{dx}x^2 = 2x$

$\frac{dy}{dx}(sinx) = cosx$

$\frac{dy}{dx}(cosx) = -sinx$

$\frac{dy}{dx}(tanx) = sec^2(x)$

$\frac{dy}{dx}(cotx) = -cosec^2(x)$

$\frac{dy}{dx}(cosec\,x) = -(cosec\,x)(cotx)$

$\frac{dy}{dx}(secx) = (secx)(tanx)$

$\frac{dy}{dx}ln(x) = \frac{1}{x}$

$\frac{dy}{dx}a^x = a^x\,loga$

$\frac{dy}{dx}x^x = x^x + ln\,x)$

$\frac{dy}{dx}e^x = e^x$

$\frac{dy}{dx}(k) = 0;\ k\ is\ any\ constant$

$\frac{dy}{dx}sin^{-1}x = \frac{1}{\sqrt{1}} - x^2$

$\frac{dy}{dx}cos^{-1} = \frac{-1}{\sqrt{1}} - x^2$

$\frac{dy}{dx}tan^{-1}x = 1/1 + x^2$

$\frac{dy}{dx}cot^{-1}x = -1/1 + x^2$

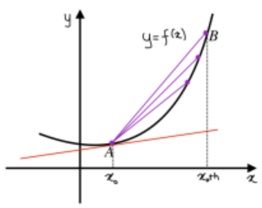
$\frac{dy}{dx}sec^{-1} = 1/IxI\sqrt{x}^2 - 1$

$\frac{dy}{dx}cosec^{-1}(x) = -\frac{1}{IxI\sqrt{x^2}} - 1$

Differentiation is the process that helps us to calculate the gradient or slope of a function at different points. It also aids in identifying changes in one variable with respect to another variable.

<u>Finding the derivative</u>

Consider the curve f(x):



As point *B* moves closer to point *A*, the gradient of chord *AB* gets closer to the gradient of the tangent to the curve at *A*.

The coordinate of *A* is (x_0, f(x_0)) and *B* is ($x_0 + h$, f($x_0 + h$)). So, the gradient of *AB* is
($x_0 + h$, f($x_0 + h$)) – (x_0, f(x_0)) / h
As *h* gets smaller, the gradient of *AB* gets closer to the gradient curve at *A*.
Hence, the gradient function, or the derivative of a curve y = f(x) is given by:

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \qquad \lim_{h\to 0} \text{ means the limit as } h \text{ tends to } 0.$$

<u>Differentiating x^n</u>

$$y = f(x) = x^n \text{ then } \frac{dy}{dx} = f'(x) = nx^{n-1} \quad \text{where } n \text{ is any real number}$$

$$y = f(x) = ax^n \text{ then } \frac{dy}{dx} = f'(x) = anx^{n-1} \quad \text{where } n \text{ is any real number and } a \text{ is a constant}$$

<u>Gradients, tangents, and normal</u>

The normal to a curve at point *A* is a straight line passing through *A* and perpendicular to the tangent line at point *A*.
For a curve y = f(x), the gradient of the tangent at point *A* with x-coordinate *a* is f'(*a*).
The equation of a tangent to the curve y = f(x) at the point with coordinates (*a*, f(*a*)) is given by:

$$y - f(a) = f'(a)(x - a)$$

So, since the gradient of the tangent at point A is f'(*a*), the gradient of a Normal at point A will be 1 / f'(*a*)
The equation of a normal to the curve y = f(x) at point *A* ≡ (*a*, f(*a*)) with gradient 1 / f'(*a*) is given by

$$y - f(a) = -\frac{1}{f'(a)}(x - a)$$