AS MATH - PURE 1

Chapter 1 - Quadratics

<u>Quadratic equations</u>

An example of a quadratic equation: $f(x) = ax^2 + bx + c$

There are two ways to solve quadratic equations:

1. *Breaking the Middle Term*

Take the equation $6x^2 + 19x + 10$

1) Find the product of the first and last term (a . c)
   $6 \times 10 = 60$

2) Find the factors of 60 in such a way that addition or subtraction of that factors is the middle term ($19x$)
   Two factors of 60 include <u>15</u> and <u>4</u> which, when added together, give 19

3) Splitting of the middle term - (15 x 4 = 60 **and** 15 + 4 = 19). Write the middle term using the sum of the two new factors, including the proper signs:
   $6x^2 + 15x + 4x + 10$

4) Group the terms to form pairs - the first two terms and the last two terms go together. Factor each pair by finding common factors:
   $3x(2x + 5) + 2(2x + 5)$

5) Factor out the shared (common) binomial parenthesis:
   $(3x + 2)(2x + 5)$

2. *Using the Quadratic Formula*

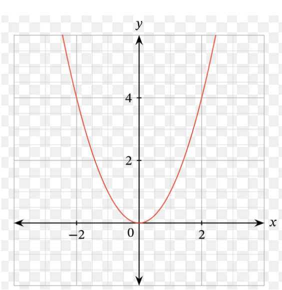
The Quadratic Formula: $-b \pm \sqrt{(b^2 - 4ac)} / (2a)$

Take the same equation $6x^2 + 19x + 10$

1) Insert the respective values in the formula
   $(-19 \pm \sqrt{19^2 - (4 \times 6)(10)}) / (2 \times 6)$

2) Use the formula to get the answer first with the minus sign then repeat with an addition sign
   $(-19 - \sqrt{19^2 - (4 \times 6)(10)}) / (2 \times 6) = 2/3$
   $(-19 + \sqrt{19^2 - (4 \times 6)(10)}) / (2 \times 6) = -5/2$

<u>The Quadratic Curve</u>

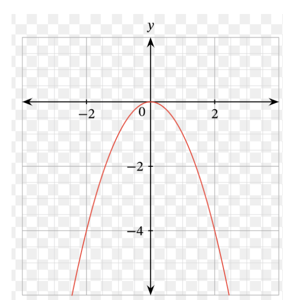


If the coefficient of x^2 is +ve then the curve is downward shaped (happy face)

If the coefficient of x^2 is -ve then the curve is upward shape (sad face)

> the coefficient is +ve happy face

the coefficient is a -ve sad face <



<u>The "Completing the Square" Method</u>

This method is used for finding the turning point of a graph and helps in plotting a parabola.
The completing the square equation is $n(x - h)^2 + k$

- ∟ If n > 0, the parabola will have a MINIMUM value (so it will be an upward-facing curve)
- ∟ If n < 0, the parabola will have a MAXIMUM value (so it will be a downward-facing curve)

In the equation, the point (h,k) is the turning point.

How to get a quartic equation into completing the square

Step 1: Write the quadratic equation as $x^2 + bx + c$. (the coefficient of x^2 needs to be 1. If not, take it as the common factor.)

Step 2: Determine half of the coefficient of x.

Step 3: Take the square of the number obtained in Step 1.

Step 4: Add and subtract the square obtained in Step 2 from the x^2 term.

Step 5: Factorize the polynomial and apply the algebraic identity $x^2 + 2xy + y^2 = (x + y)^2$ (or) x^2, $2xy + y^2 = (x + y)^2$ to complete the square.

Quadratic Inequalities

There are 3 types of quadratic inequalities:

- $ax^2 + bx + c < 0$
- $ax^2 + bx + c > 0$
- $ax^2 + bx + c = 0$

To find the discriminant, use the formula $b^2 - 4ac$

- $ax^2 + bx + c < 0$ has no real roots
- $ax^2 + bx + c > 0$ has two distinct real roots (distinct = different)
- $ax^2 + bx + c = 0$ has one real root

Chapter 2 - Functions and Transformations

A function is a relation that maps inputs to outputs.

- The function has both y and x values
- The x value imputed in the function is called the domain
- The y value output by the function is called the range

Composite functions are formed when you combined two or more functions, for example:

$$f(x) = 3x + 2 \qquad g(x) = 7 - x$$

Therefore, fg(x) would be the function g entered as an input of x in function f and simplified

$$f(x) = 3(7 - x) + 2$$

The function inverse is the reflection of function in the line $y = x$ changing all x coordinates to y coordinates and y to x, thus switching the range and domain of a function.

How to find the inverse function

Make f(x) as y and then make x the subject of the equation then replace y as x, for example:

$$f(x) = 3x + 2$$

$$y = 3x + 2$$
$$y - 2 = 3x$$
$$(y - 2)/3 = x$$

So the inverse of f(x) is (x-2)/3

Transformations of the functions

- For any function f(x), the graph of $y = f(x) + a$ can be obtained from the graph of $y = f(x)$ by translating it through a unit in the positive y direction.
- For any function f(x), the graph of $y = f(x - a)$ can be obtained from the graph of $y = f(x)$ by translating it through a unit in the positive x direction.
- For any function f(x), the graph of $y = f(x - s) + t$ can be obtained from the graph of $y = f(x)$ by translating it through s units in the positive x direction and t units in the positive y direction.
- For any function f(x), and any positive value of a, the graph of $y = af(x)$ can be obtained from the graph of $y = f(x)$ by a stretch of the scale factor a parallel to the y-axis
- For any function f(x), and any positive value of a, the graph of $y = f(ax)$ can be obtained from the graph of $y = f(x)$ by a stretch of scale factor 1 parallel to the x-axis.

Chapter 3 - Coordinate Geometry

The chapter relating to finding line segments, gradients, and midpoints of those line segments

Formulas

General Formula of a Line	$Ax + By + C = 0$
Slope Intercept Formula of a Line	$y = mx + c$
Point-Slope Formula	$y - y_1 = m(x - x_1)$
The slope of a Line Using Coordinates	$m = \Delta y/\Delta y = (y_2 - y_1)/(x_2 - x_1)$
The slope of a Line Using a General Equation	$m = -(A/B)$
Intercept-Intercept Formula	$x/a + y/b = 1$
Distance Formula	$\|P1P2\| = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$
For Parallel Lines	m1 = m2
For Perpendicular Lines	m1m2 = -1
Midpoint Formula	$M(x, y) = [\frac{1}{2}(x_1 + x_2), \frac{1}{2}(y_1 + y_2)]$

Chapter 4 - Circular measure

This chapter is related to finding circle dimensions, including area and radius sector length.
All the questions are in radian mode.

360 degrees = 2π radians

Formulas

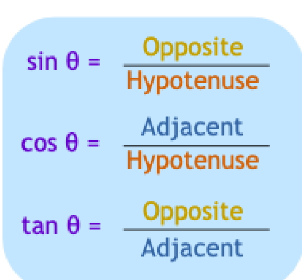
$$S = r\theta$$

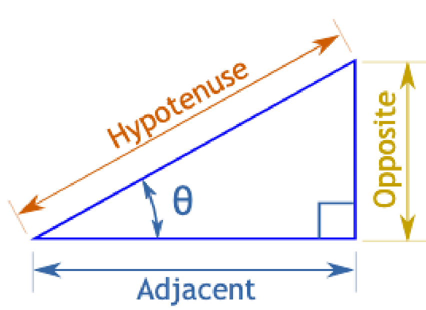
S is the sector length
r is the radius
θ is the angle (in radians)

Area of a Sector - Formula

$Area_{circular\ sector} = \frac{1}{2}r^2\theta$ For the perimeter of a sector add the arc length and radius two times.

Chapter 5 - Trigonometry





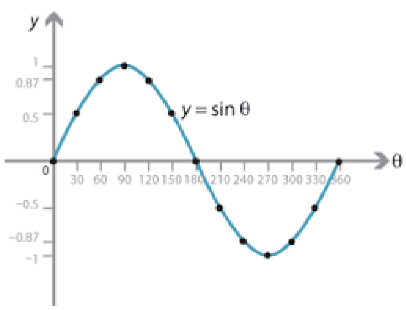
There are 3 main trigonometric functions:
- Sin
- Cos
- Tan

Trigonometric identities

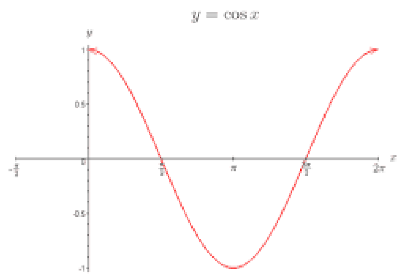
- Sin θ = 1/Cosec θ *or* Cosec θ = 1/Sin θ
- Cos θ = 1/Sec θ *or* Sec θ = 1/Cos θ
- Tan θ = 1/Cot θ *or* Cot θ = 1/Tan θ
- $\sin^2 a + \cos^2 a = 1$
- $1 + \tan^2 a = \sec^2 a$
- $\mathrm{cosec}^2\, a = 1 + \cot^2 a$

Trigonometric graphs - these can be both in degrees and radians

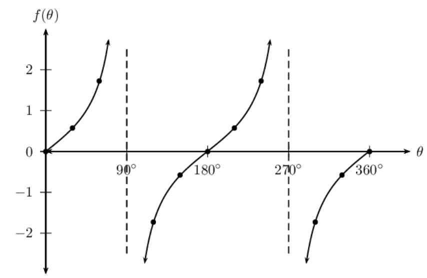
Sin Graph



Cos Graph



Tan Graph



Chapter 6 - Binomial Expansion

- The total number of terms in the expansion of $(x+y)$n is (n+1)
- The sum of exponents of x and y is always n.
- nC0, nC1, nC2, … .., nCn are called binomial coefficients and are represented by C0, C1, C2, ….., Cn
- The binomial coefficients, which are equidistant from the beginning and the ending, are equal, i.e., nC0 = nCn, nC1 = nCn-1, nC2 = nCn-2,….. Etc.

Binomial formula

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

$$= \sum_{k=0}^{n} \binom{n}{k} x^{k} y^{n-k}$$

Examples

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$a^2 - b^2 = (a-b)(a+b)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Chapter 7 - AP GP

AP = Arithmetic Progression
(AP) is a sequence of numbers in order, in which the difference between any two consecutive numbers is a constant value

- ∟ First term (a)
- ∟ Common difference (d)
- ∟ nth Term (an)
- ∟ Sum of the first n terms (Sn)

General formula = **a + (n – 1) d**
General term = **an = a + (n − 1) × d**
Sum of terms = **Sn = n/2[2a + (n − 1) × d]**

GP = Geometric Progression
(GP) is a type of sequence where each succeeding term is produced by multiplying each preceding term by a fixed number, which is called a common ratio

- ∟ Three non-zero terms – a, b, c – are in GP only if b2 = ac
- ∟ In a GP, three consecutive terms can be taken as a/r, a, ar
- ∟ Four consecutive terms can be taken as a/r^3, a/r, ar, ar^3
- ∟ Five consecutive terms can be taken as a/r^2, a/r, a, ar, ar^2
- ∟ In a finite GP, the product of the terms equidistant from the beginning and the end is the same. That means, t1.tn = t2. tn-1 = t3. tn-2 = ...
- ∟ If each term of a GP is multiplied or divided by a non-zero constant, then the resulting sequence is also a GP with the same standard ratio
- ∟ The product and quotient of two GPs is again a GP
- ∟ If each term of a GP is raised to the power by the same non-zero quantity, the resultant sequence is also a GP

General formula = **ar^{n-1}**
General term formula = **an = tn = ar^{n-1}**
Sum of terms formula = **Sn = $a[(r^n – 1)/(r – 1)]$ if r ≠ 1 and r > 1**
Sum to infinity formula = **S∞=a1/(1-r)**

Chapter 8 - Differentiation

Rules of Differentiation

Differentiation of a **scalar multiple** of a function	$\frac{d}{dx}(ay) = a\frac{dy}{dx}$
Differentiation of the **sum/difference** of a function	$\frac{d}{dx}(y_1 \pm y_2) = \frac{dy_1}{dx} \pm \frac{dy_2}{dx}$
Differentiation of **Constant Function** y = c, where c is a constant	$\frac{dy}{dx} = 0$
Differentiation of a **Power Function,** where n is a real number	$\frac{dy}{dx} = nx^{n-1}$

$\frac{dy}{dx}x^2 = 2x$

$\frac{dy}{dx}(sinx) = cosx$

$\frac{dy}{dx}(cosx) = -sinx$

$\frac{dy}{dx}(tanx) = sec^2(x)$

$\frac{dy}{dx}(cotx) = -cosec^2(x)$

$\frac{dy}{dx}(cosec\,x) = -(cosec\,x)(cotx)$

$\frac{dy}{dx}(secx) = (secx)(tanx)$

$\frac{dy}{dx}ln(x) = \frac{1}{x}$

$\frac{dy}{dx}a^x = a^x loga$

$\frac{dy}{dx}x^x = x^x + ln\,x)$

$\frac{dy}{dx}e^x = e^x$

$\frac{dy}{dx}(k) = 0;\ k\ is\ any\ constant$

$\frac{dy}{dx}sin^{-1}x = \frac{1}{\sqrt{1}} - x^2$

$\frac{dy}{dx}cos^{-1} = \frac{-1}{\sqrt{1}} - x^2$

$\frac{dy}{dx}tan^{-1}x = 1/1 + x^2$

$\frac{dy}{dx}cot^{-1}x = -1/1 + x^2$

$\frac{dy}{dx}sec^{-1} = 1/IxI\sqrt{x}^2 - 1$

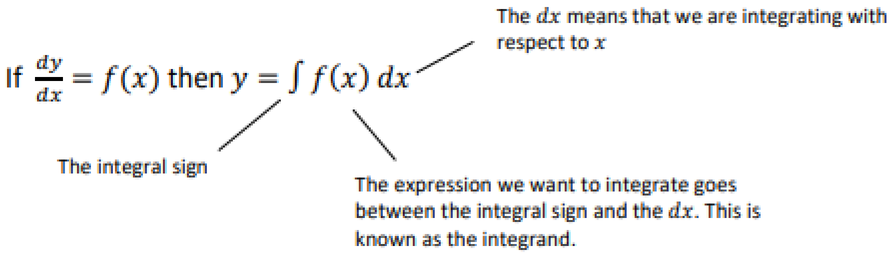
$\frac{dy}{dx}cosec^{-1}(x) = -\frac{1}{IxI\sqrt{x^2}} - 1$

Chapter 9 - Integration

Integration is the reverse of differentiation. We can use integration to find areas bounded between a curve and the coordinate axes.

Notation

The $\int$ symbol is used to represent integration. Since integration is the reverse of differentiation, we know that:



Indefinite integrals

Here, you need to integrate functions of the form x^n, where n is a constant and $n \neq -1$. To integrate functions of this form, you can use the following:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

The "+c" is known as the constant of integration. To see why we must add this constant to our result, consider these functions:

$$y = x^2 + 2$$
$$y = x^2$$
$$y = x^2 - 9$$

If we differentiate the above functions, the result is the same: $dy/dx = 2x$ because the constant term disappears upon differentiation. However, since integration is the reverse of differentiation, we should be able to integrate 2x and get back to whichever of those functions we started off with. To allow for this, we have to add the unknown constant of integration, *c*, to the end result. This process is known as indefinite integration.

Definite integrals

A definite integral is one where the integral is bounded between two limits. The main difference between a definite integral and an indefinite integral is that the former will yield a numerical value while the latter will yield a function. To calculate a definite integral:

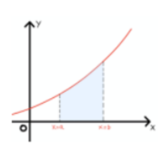
$$\int_a^b f'(x)\, dx = [f(x)]_a^b = f(b) - f(a)$$

Finding Areas

You can use definite integration to find the area bounded between a curve and the x-axis (areas under the line of the curve).

The area between a curve y = f(x), the lines x = a, x = b, and the x-axis is given by:

$$Area = \int_a^b f(x)\, dx$$



Areas under the x-axis

When integrating over an interval where the curve is below the x-axis, the resultant area will be negative. Therefore, extra care must be taken when finding the areas under curves which are not positive.

★ When integrating over an interval where the curve is both above and below the x-axis, you should split the integral up into separate regions where the function is strictly positive or negative in each.

Integration formulas:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq 1$$

$$\int dx = x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$\int \frac{dx}{1+x^2} = -\cot^{-1} x + C$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{dx}{x} = \log|x| + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$