Chapter 6 - Binomial Expansion

- The total number of terms in the expansion of $(x+y)^n$ is (n+1)
- The sum of exponents of x and y is always n.
- nC0, nC1, nC2, … .., nCn are called binomial coefficients and are represented by C0, C1, C2, ….., Cn
- The binomial coefficients, which are equidistant from the beginning and the ending, are equal, i.e., nC0 = nCn, nC1 = nCn-1, nC2 = nCn-2,….. Etc.

Binomial formula

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$
$$= \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Examples

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$a^2 - b^2 = (a-b)(a+b)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Pascal's Triangle

Knowing that $(a + b)^2 = a^2 + 2ab + b^2$, you should be able to work out that $(a + b)^3 = a^3 + 3a^2b + 3b^2a + b^3$.

$(a + b)^1 = a + b$.

so $(a + b)^1 = \quad a + b$

$(a + b)^2 = \quad a^2 + 2ab + b^2$

$(a + b)^3 = a^3 + 3a^2b + 3b^2a + b^3$

Note that the coefficients of a and b are:

1 1
1 2 1
1 3 3 1

If you continued expanding the brackets for higher powers, you would find that the sequence continues:

1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

This sequence is known as **Pascal's triangle**. Each of the numbers is found by adding together the two numbers directly above it.

The 20 in the last line is found by adding together 10 and 10. Each of the 10s in the line above is found by adding together a 6 and a 4, and so on.

Therefore, it is possible to expand (a + b) to any whole number power by knowing Pascal"s triangle.