Chapter 4 - Circular measure

This chapter is related to finding circle dimensions, including area and radius sector length.
All the questions are in radian mode.

360 degrees = 2π radians

Formulas

S = rθ

S is the sector length
r is the radius
θ is the angle (in radians)

Area of a Sector - Formula

$Area_{circular\ sector} = \frac{1}{2}r^2\theta$ For the perimeter of a sector add the arc length and radius two times.

The Equation of a Circle

The equation of any circle with center (a,b) and radius *r* is

$$(x - a)^2 + (y - b)^2 = r^2$$

As all the points on the circumference have the same distance to its center, the radius. The question of the circle is derived using the Pythagoras theorem.

You can also come across circles that have equations that come in the form of $ax^2 + by^2 + cx + dy + e = 0$
The equations that come in this form are just $(x - a)^2 + (y - b)^2 = r^2$ multiplied out and simplified. To get from the form $ax^2 + by^2 + cx + dy + e = 0$ to $(x - a)^2 + (y - b)^2 = r^2$, you use the completing square method.

Intersections of straight lines and circle

You can use similar methods to find intersection points of a straight line and a circle as you did with a straight line and another straight line or a curve.

There can either be one, two, or zero intersection points between a straight line and a circle.

Example: *the line l_1 with the equation $y = 2x + 3$ meets the circle with the equation $(x - 3)^2 + (y - 4)^2 = 9$ at two distinct points. Find the x-coordinates of these two intersection points.*

In order to find the solution, we can solve the equations simultaneously.

$$(x-3)^2 + (y-4)^2 = 9$$
$$y = 2x + 3$$

$$(x-3)^2 + (2x+3-4)^2 = 9$$
$$(x-3)^2 + (2x-1)^2 = 9$$
$$x^2 - 6x + 9 + 4x^2 - 4x + 1 = 9$$
$$5x^2 - 10x + 1 = 0$$

We can use the quadratic equation to solve this.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 5 \times 1}}{2 \times 5}$$
$$x = \frac{5 + 2\sqrt{5}}{5} \quad and\ x = \frac{5 - 2\sqrt{5}}{5}$$